Correlated Noise Estimation and Error Correction in Parallel Storage Channels

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***Abstract*—The evolution of new technologies frequently results to the need of formulating old problems in new way. The design of MEMS-based storage systems that use multiple, simultaneously accessed channels, has to confront, among others, the problem of correlated noise in different channels. This work presents a method that exploits the inherent parallelism found in communication/storage systems that use multiple, simultaneously accessed channels in order to estimate the correlated error vector and to improve the data decoding process. The method uses the parallelism of the multiple channels to flag symbols not only as erasures using the error locations revealed by the initial errors-only decoding attempt, but to estimate the introduced channel noise vector and provide soft information to the demodulators of all channels. Numerical results demonstrate the performance improvement that is achieved by the proposed method, thus improving the reliability of such systems/devices.**

I. INTRODUCTION

In recent years, due to the constantly increasing need for higher data rates in storage systems, the use of multiple, simultaneously operating channels has been introduced [1]. These channels experience various noise sources from different physical phenomena and some of these noise sources are independent to each other, but in some cases, the same physical mechanism affects all channels, thus resulting to correlated introduced noise. Although this work was inspired by probe-based storage devices with multiple parallel channels [2], the following analysis and the proposed method can be applied to any communication system, baseband or broadband, that uses multiple parallel channels and is affected by correlated noise sources.

In general, the architecture of such a system with parallel channels is shown in Fig.1. In each channel, a block of user data, along with suitable redundancy information imposed by the error correction scheme used to achieve the application-specific reliability, is modulated according to the channel characteristics and transmitted over the channels. Different data blocks are transmitted concurrently, each one over a different channel. Depending on the communication medium, the channels are subject to various noise sources and distortions. In most cases, the noise sources are considered independent and techniques, like interleaving, are used to spread bursts of errors to multiple codewords and/or channels. In other cases, a common noise source, like an external mechanical shock to a micro-mechanical system with multiple equidistant probes, results to equal displacement and the introduced noise can be correlated.

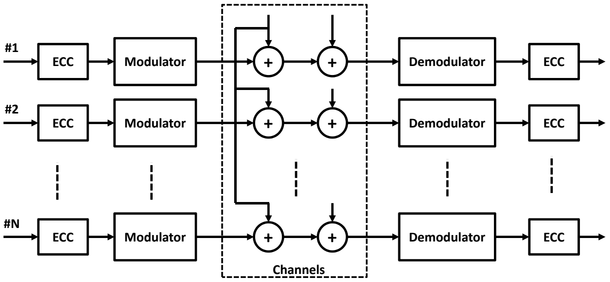


Fig. 1. System with multiple parallel channels.

In the case shown in Fig.1, we consider two sources of noise at the system level. A common noise source affects all channels (a practical example of such a system is presented in Section II), while random, uncorrelated noise sources affect independently all channels.

For confronting the introduced errors, several coding schemes have been proposed and studied for such systems. Most commonly, BCH or Reed-Solomon codes are used, which combine random and burst error correction capabilities, along with affordable circuits complexity [3], [4]. When the parallel channels are statistically independent, there is no correlation among the errors that appear on different channels due to the additive noise sources. However, when the common noise source is taken into account, which affects all channels with the same statistical characteristics, and depending on the number of channels and the duration of the noise effect, a great number of burst errors appear in all codewords and the errors are correlated. BCH/RS codes that can correct up to t errors, can be extended to correct up to $2t$ errors as long as up to t error positions are known, called erasures. In [2], a heuristic approach for erasure estimation and error correction in storage devices that use multiple, simultaneously accessed parallel fields, when they are affected by burst errors caused by external disturbances, was presented. That algorithm exploits the parallelism of the multiple fields and the error locations revealed by the initial errors-only decoding attempt and identifies symbols as erasures.

In this work, we propose an extended version of this method in order, not only to determine erasures for the BCH/RS decoders, but also to provide an accurate estimate of the common noise vector. In this method, a new demodulation-decoding round is initiated for the non-decodable codewords, where each channel demodulator uses the estimated noise vector to subtract an estimate of the common noise introduced to the channels, thus providing better estimates of the received symbols and enhancing the BCH/RS successful decoding capability.

Section II presents concisely the two basic technologies initiated this work and describes a real system, where this type of noise is introduced. Section III discusses in details the proposed method and presents the proposed receiver's architecture. Section IV presents the method for identifying potential error locations as erasures, while Section V analyzes the method for providing an estimate of the noise vector introduced in the channels. Finally, Section VI presents numerical results that justify the use of both methods for improved system reliability.

II. PCM/PROBE SYSTEMS WITH PARALLEL CHANNELS

This section presents concisely the two basic technologies related with the current work, namely probe-based micro-systems and Phase-change memory (PCM).

Scanning probes with nanometer-sharp tips are used extensively for storage applications with high storage density [1]. In most ultra-high-density probe-based data storage systems, information is stored in the form of nano-marks on thin polymer films [5]. Each probe acts as an independent modulator/transmitter during the write process and as an independent demodulator/receiver during read. Although this technique was initially used on thin polymer films with thermo-mechanical sensing, the development of new PCM materials and nano-scale conductive probes allows the application of these techniques to more advanced systems, with the capability to store multiple bits per storage mark, a technique similar to multi-level cell programming in non-volatile memories (NVM) and to M-ary transmission systems.

NVM is considered the most promising technology to provide cost- and power-effective solutions for next generation storage systems [6]. Although current systems are based on NAND Flash memory, which scales to 2x nm and offers increased read/write bandwidth and memory capacity, PCM may become the next dominant technology of resistive non-volatile memories for storage devices and systems. PCM is based on changing the crystalline/amorphous states of nano-scale cells resulting to highly different resistances. By using iterative programming strategies PCM offers multi-bit operation per storage element [7], and this capability can be exploited also on PCM/probe-based storage systems. The basic model of the readback signal generated when a data pattern stored in a PCM surface is scanned was presented in [5]. The presented two-dimensional model was based on Finite Element Method analysis and was used to derive the basic waveform of the readback signal from an amorphous mark, for different geometric and physical configurations of the storage system. Although this model represents a two-levels signal, it can be extended to model multiple level signals, as the ones studied in this work.

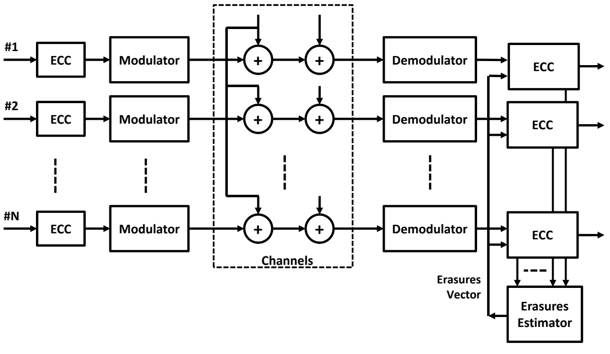
The work presented in this paper was initiated by the design of a PCM/probe-based MEMS accelerometer that uses multiple parallel probes and senses pre-stored data patterns using 4-levels PCM marks. The basic considerations have been generalized for a system, either communications or storage, with multiple parallel channels with correlated and independent noise sources. Although such a system can be considered as a baseband-like communication system, the presented analysis can also be applied to broad-band communication channels with two-dimensional constellation. Such a constellation is used in the illustrative example presented in Section V for explaining the proposed method.

III. CORRELATED ERROR VECTOR ESTIMATION

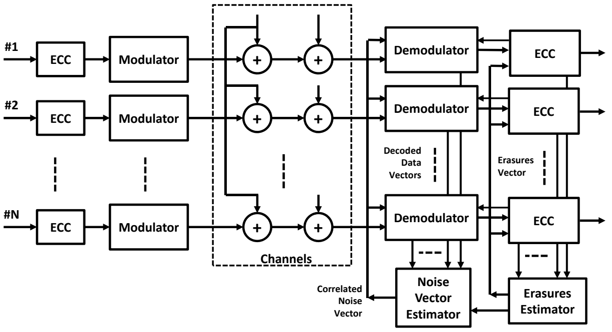
As shown in Fig.1, the use of the typical error correction approach is not the most efficient in probe-based systems, since it does not exploit the inherent parallelism and the common noise sources met in such systems. The method introduced in [2] exploits the parallelism of the multiple channels and the error locations revealed by the initial errors-only decoding attempt and identifies symbols as erasures, thus increasing the error correction capability of the used codes.

In this work, we propose an extended version of this method in order, not only to determine erasures for the BCH/RS decoders, when the parallel channels are affected by both random and correlated noise, as shown in Fig. 2(a), but also to reconsider the outputs of the demapper circuits by subtracting the estimated noise vector. This method can be applied when the standard decoding procedure succeeds in decoding at least a small number of codewords, thus revealing certain error locations and can be used to enhance the error correction capability without increasing the total overhead information. We also present the respective algorithm, where the decoded codewords are used to provide an estimate of the noise vector added to the transmitted data due to the noise sources that affect all channels. As shown in Fig. 2(b), a new demodulation-decoding round is initiated for the non-decodable codewords, where each channel demodulator uses the estimated noise vector to subtract part of the introduced noise, thus providing better estimates of the received symbols and enhancing the total decoding capability. Furthermore, a combination of both approaches, correlated noise vector and erasures, can be employed.

Fig. 3 presents in details the proposed method and the respective receiver's architecture. Starting with the ECC decoder, during the typical decoding process, the demapper output values are kept in a local memory and statistics regarding the number of codewords correctly decoded and the number of errors found are collected. As proposed in [2], if there is at least one correct codeword, then an additional round of decoding is initiated. The statistics from all the corrected codewords are passed to the erasure estimator block, which



(a) Erasure estimation



(b) Noise vector estimation.

Fig. 2. Methods for enhancing the system reliability when data, in multiple but not all channels, are decoded correctly by the typical ECC decoders.

then produces a vector with the bit locations that are mostly affected with errors. These bit locations are flagged as erasures and an iteration of errors-and-erasures decoding starts for the non-decodable codewords.

IV. DECODING WITH ERASURES

BCH decoding with erasures information has been proven to provide enhanced reliability compared to typical, errors-only decoding on burst error channels. However, this advanced decoding capability is mostly affected by the reliability to determine the symbols that will be flagged as erasures, because mistaken assumptions can deteriorate the performance of the decoder [8]. We propose a method that assigns an erasure probability to every bit location of a codeword non-decoded by the errors-only decoder, based on the information of the erroneous and correct bits that were detected in the same bit location in other simultaneously operating channels.

An illustrative example of how the proposed erasures-based method is applied is given in Fig. 4. The system includes 4 channels and the data are organized in BCH(n, k) codewords. Due to the correlated noise source, in this example, we assume that only one codeword is decodable by the BCH decoder, while the other 3 codewords cannot be decoded correctly. Column j denotes the position of the bits that are transmitted concurrently on all channels. Then c_j corresponds to the number of bits known to be correct as detected by the first decoding attempt, while e_j corresponds to the number of bits known to be in error in position j. We use the erroneous bit proportion $Pe_j = e_j/(c_j + e_j)$ as an estimate of the probability that the bit of unknown state in the same position j is in error. This estimate is called *erasure probability*. The symbols with the highest erasure probabilities are flagged as erasures and an additional errors-and-erasures decoding step is performed. The method starts with an initial number of erasures, usually 2, and is executed repetitively by flagging additional symbols as erasures, until either all codewords are decoded successfully, or a maximum allowed number of erasures is reached. Algorithm 1 gives a detailed description of the proposed method.

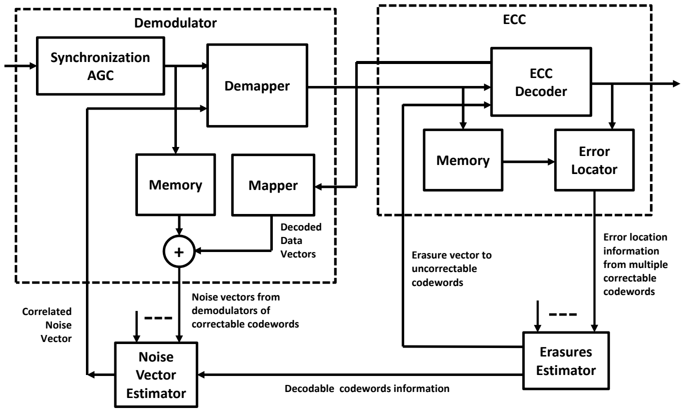


Fig. 3. Correlated Noise Vector Estimation.

Algorithm 1 Erasure Estimation Algorithm

```
for all j do
    $Pe_j \leftarrow \frac{e_j}{c_j+e_j}$
end for
repeat
    do errors-and-erasures decoding using as erasures the symbols with the highest $Pe_j$
    if Codeword Decoding Success then SUCCESS
    else
        if at least 1 new Codeword Decoding Success then recalculate $Pe_j$
            if erasures $\neq$ max then erasures++
            end if
        else
            if erasures $\neq$ max then erasures++
            else FAILURE
            end if
        end if
    end if
until SUCCESS or FAILURE
```

When BCH codes are used, the circuits of the typical decoder can also be used for the errors-and-erasures decoding. The decoding is accomplished in two steps. First, the erased positions are replaced with $0'$s and the new data vector is decoded using the standard BCH decoding algorithm. Next, the erased positions are replaced with 1's and the

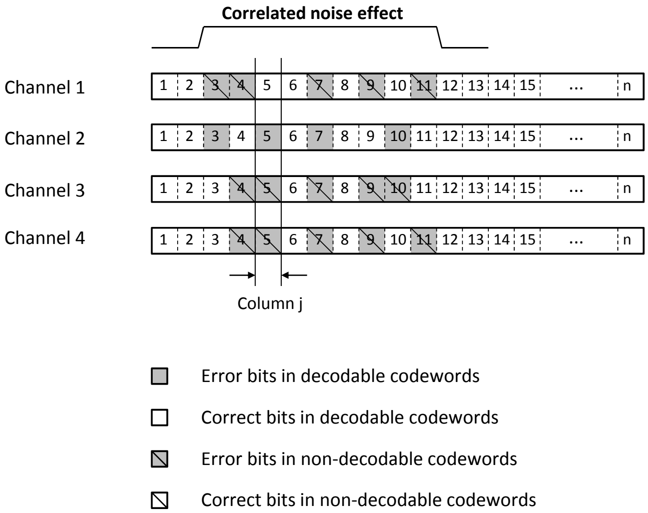


Fig. 4. An illustrative example of the errors and erasures decoding algorithm.

resulting vector is decoded following the same procedure. The decodings result in two codewords, and if the estimations are valid, at least one of the two recovered datawords is correct [4].

V. ERROR VECTOR ESTIMATION

In the case where a common noise source affects all channels, then the error vector that is added to the transmitted symbols is the same for all channels. More precisely, the correlated noise introduced in each channel is composed of a common component (the mean value of all correlated noises) and a variation modeled as random noise. In our model, the random noise component of the correlated noise is considered as part of the independent random noise. This, along with the fact that usually the same random, uncorrelated noise sources affect independently all channels, means that, whenever the initial decoding attempt does not decode successfully all codewords in all channels, but manages to decode few of them, the information regarding those channels can be used to estimate the correlated noise that affected the whole system.

The idea is illustrated with the example presented in Fig. 5. The system of this example includes 4 channels that use 4-QAM modulation for transmission. In Fig. 5(i), the 4 different symbols that are transmitted concurrently in each channel are shown, along with the noise that affects them. The original symbols are represented with the black dots, while the actually received ones with the red dots. The correlated noise vector is depicted with a dark red arrow, the random noise with a green arrow and the total noise vector is the blue arrow. We assume that the codewords in channels 1, 2 and 4 are successfully decoded by the decoder, while the codeword in channel 3 is non-decodable.

For each symbol j in a corrected codeword i, the error vector $e_i(j)$ can be extracted. For that purpose, symbol mapping is applied to the corrected data and the symbol sequence tx_i that was originally transmitted is regenerated. Then, the error vector is calculated: $e_i = rx_i - tx_i$. Using this information, for each symbol j in a non-decodable codeword, a mean error vector $\bar{e}(j)$ can be calculated as shown in Fig. 5(ii). This is an estimate of the total noise vector that affects the system. Fig. 5(iii) shows that if this noise estimate is subtracted by the respective demapper output of the initially uncorrected symbol, then it may become correctable. So, an additional round of demapping the new symbol sequences rx'_z for each non-decodable codeword z, followed by BCH decoding, can lead to better symbol detection and data correction. The proposed method is described in detail in Algorithm 2.

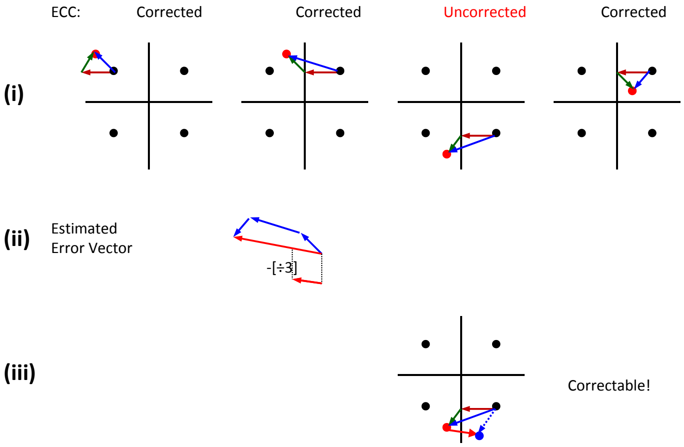


Fig. 5. Error vector estimation example.

VI. NUMERICAL RESULTS

We consider a system with $N = 8$ parallel channels, 4-PAM modulation for transmission and BCH(511,456) code for ECC. That means that the BCH code can correct up to $t = 6$ errors. We study the case where, while the N different codewords are transmitted over the N parallel channels concurrently, a common noise source affects all channels for a certain duration, leading to a burst of errors in all channels, and at the same time random noise sources affect all channels independently. We are interested in the cases where, while some codewords are not decodable by the typical BCH decoder, at least one codeword is decoded successfully. In these cases, we initially apply the proposed method to estimate the locations of errors in the not-decoded codewords and extend the error correction capability of the BCH code from t errors to at most $2t$ errors, using the errors-and-erasures approach.

As a next step, we estimate the error vector that was added to the channels due to the various noise sources and re-initiate the demodulation-decoding procedure with better estimates of the received symbols. Fig. 6 shows the probability distributions regarding the mean number of codewords that cannot be decoded and the mean number of errors that appear in a codeword for various scenarios, regarding the statistical characteristics of the random and the correlated noise that affect the system. The total SNR that affects the system for the various scenarios is given in Table I.

Algorithm 2 Noise Vector Estimation Algorithm

```
for all correctly decoded codewords $i$ do
    Use the mapper to produce the transmitted symbol sequence $tx_i$
    $ei = tx_i - rx_i$
    $\bar{e} = \sum(e_i)/\sum(i)$
end for
repeat
    for all non-decodable codewords $z$ do
        $rx'_z = rx_z - \bar{e}$
        Perform demapping-decoding procedure
    end for
    if Codeword Decoding Success then SUCCESS
    else
        if at least 1 new Codeword Decoding Success then recalculate $\bar{e}$
        else FAILURE
        end if
    end if
until SUCCESS or FAILURE
```

TABLE I
TOTAL SNR DUE TO RANDOM AND CORRELATED NOISE SOURCES

Correlated SNR (dB) \ Random SNR (dB)	19	20	21	22
12	12.81	13.03	13.21	13.36
14	14.24	14.54	14.81	15.03
16	15.46	15.88	16.24	16.54
18	16.46	16.99	17.46	17.88

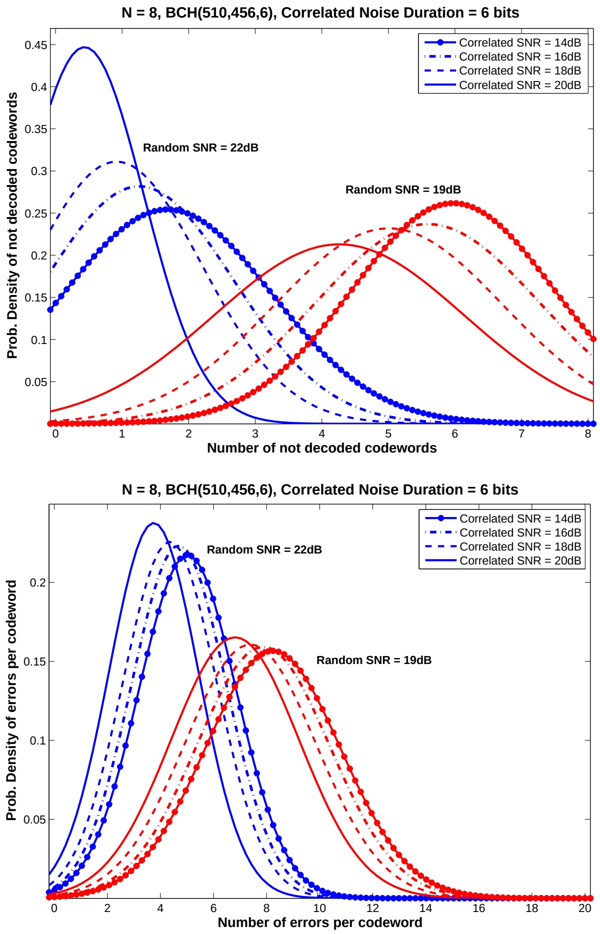


Fig. 6. Probability distributions of the number of errors per codeword and the number of not decoded codewords, when random and correlated noise sources affect a system with $N = 8$ channels and BCH(510,456,6) code.

Since the correlated noise source causes the appearance of burst errors in all channels, we assume that its mean duration is equal to the error correction capability of the BCH code, i.e. 6 bits. According to these results, there is a great chance that the proposed algorithms can be applied, since there is a large probability that at least one codeword can be decoded correctly by the initial errors-only decoding attempt. Additionally, for lower levels of the random noise, we expect a small number of non-decodable codewords, while for higher levels, almost all codewords are affected by more that t errors.

It has to be noted that regarding the errors-and-erasures approach, it can lead to a successful decoding only if the total number of errors in the codewords are less that $2t$. But Fig. 6 shows that even small increases to the noise level of the random sources, may result in the occurrence of more than $2t$ errors in the codewords, which will affect the performance of the algorithm.

Fig. 7 gives the system's reliability for various noise scenarios when only the typical BCH decoder is used. Fig. 8 shows the improvement that is observed when the two other decoding methods are applied. The blue lines denote the initial reliability, the red lines correspond to the reliability when the errors-and-erasures approach is used and the green lines to the reliability when the Error Vector Estimation method is used. Both methods are applied if at least one codeword has been corrected by the initial decoding procedure.

The results show that both methods improve significantly the reliability of the system and lead to increasingly better results as the levels of the random noise sources decrease. For example, for a random SNR od 22dB, both methods result to almost total success for all values of the correlated SNR. This can be explained by the fact that, as shown in Fig. 6, for higher values of random noise, there is a larger number of correctable codewords at the initial decoding attempt. So, the algorithm has more accurate information and is able to produce more reliable estimates of the erasure locations and the channel error vector accordingly.

It can also be observed that for higher levels of random noise, the Error Vector Estimation method gives better results than the Errors-and-Erasures method. This is explained by the fact that the application of the Error Vector Estimation method decreases the noise, and thus increases the SNR, of the regenerated signal, thus decreases slightly the raw error probability. When the dominant error source is the correlated noise, the erasure flagging process leads to highly reliable estimations and surpasses the performance of the Error

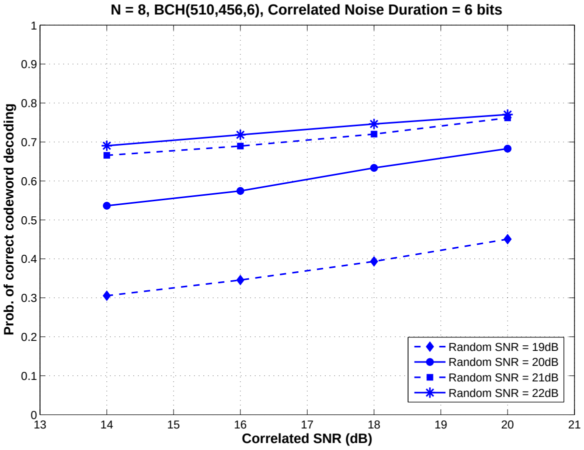


Fig. 7. Probability of successful codeword decoding for various noise scenarios with different statistical characteristics.

Vector Estimation. Finally, Fig. 9 shows the effect of the duration of the correlated noise source, on the performance of the proposed algorithms. The performance of both methods deteriorates as the correlated noise duration increases, but they still lead to a significant improvement in the device reliability comparing to the conventional decoding process. As the mean duration gets larger than the error correction capability $t = 6$ of the BCH code, the performance of the Errors-and-Erasures approach worsens, since there is a greater probability that more than $2t$ errors appear in the codewords, making them uncorrectable by the errors-and-erasures BCH decoder.

VII. CONCLUSIONS

The exploitation of the inherent parallelism found in various communications/storage systems, like the MEMS probe-based devices, may result to improved reliability without increasing the introduced overhead. This paper presents such a method that estimates the correlated noise vector and uses this information to increase the total SNR. Combining the above mentioned technique with reliable erasures-based decoding results to even further improved performance.

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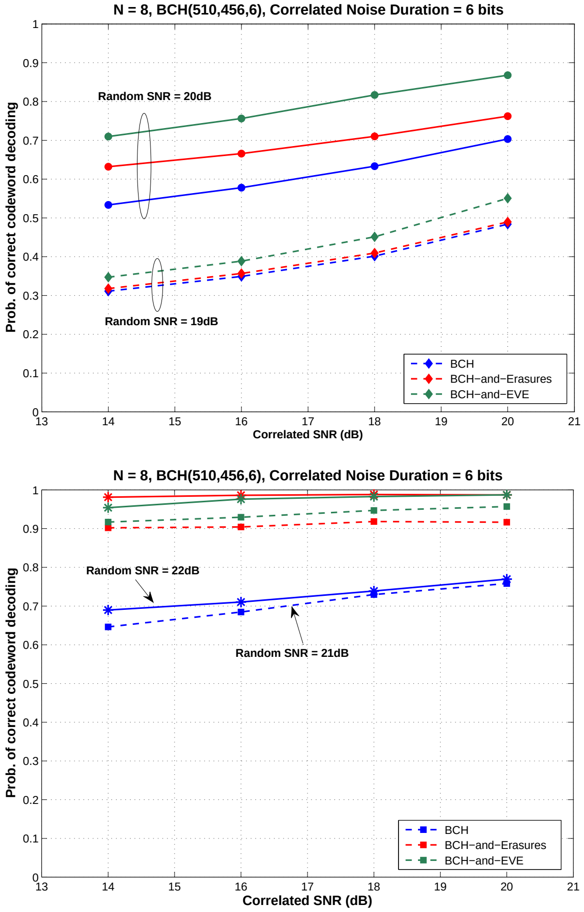

Fig. 8. Successful codeword decoding probability for various noise scenarios with different statistical characteristics.

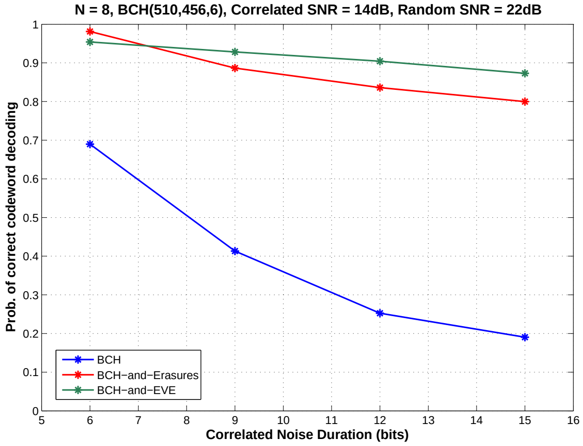

Fig. 9. The effect of the correlated noise duration.

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